Vocabulary

The table contains important vocabulary terms from Chapter 2. As you work through the chapter, fill in the page number, definition, and a clarifying example.

Term	Page	Definition	Clarifying Example
biconditional statement			
conclusion			
conditional statement			
conjecture			
contrapositive			
counterexample			
deductive reasoning			
hypothesis			

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CHAPTER 2

Vocabulary

The table contains important vocabulary terms from Chapter 2. As you work through the chapter, fill in the page number, definition, and a clarifying example.

Term	Page	Definition	Clarifying Example
biconditional statement	96	A statement that can be written in the form "p if and only if q."	A figure is a triangle if and only if it is a three-sided polygon.
conclusion	81	The part of a conditional statement following the word then.	If $x + 1 = 5$, then $\underbrace{x = 4}$. Conclusion
conditional statement	81	A statement that can be written in the form "if p, then q," where p is the hypothesis and q is the conclusion.	If $x + 1 = 5$, then $x = 4$. hypothesis conclusion
conjecture	74	A statement that is believed to be true.	A sequence begins with the terms 2, 4, 6, 8, 10. A reasonable conjecture is that the next term in the sequence is 12.
contrapositive	83	The statement formed by both exchanging and negating the hypothesis and conclusion of a conditional statement.	Statement: If $n + 1 = 3$, then $n = 2$. Contrapositive: If $n \neq 2$, then $n + 1 \neq 3$.
counterexample	75	An example that proves that a conjecture or statement is false.	Statement: All months have 30 days. Counterexample: October has 31 days.
deductive reasoning	88	The process of using logic to draw conclusions.	All squares are rectangles. *ABCD* is a square. Therefore *ABCD* is a rectangle.
hypothesis	81	The part of a conditional statement following the word if.	If $x + 1 = 5$, then $x = 4$. Hypothesis

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Term	Page	Definition	Clarifying Example
inductive reasoning			
logically equivalent statements			
negation			
polygon			
quadrilateral			
theorem			
triangle			
truth value			

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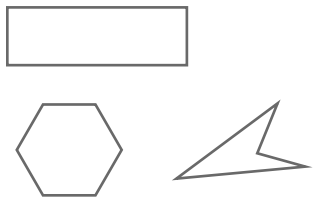
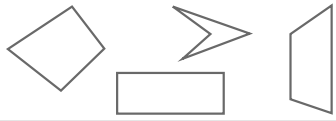
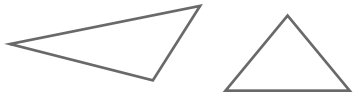
Term	Page	Definition	Clarifying Example
inductive reasoning	74	The process of reasoning that a rule or statement is true because specific cases are true.	3, 6, 9, 12, 15, … Multiples of 3 make up the pattern. The next multiple is 18.
logically equivalent statements	83	Statements that have the same truth value.	If $m\angle A = 67°$, then $\angle A$ is acute. If $\angle A$ is not acute, then $m\angle A \neq 67°$.
negation	82	The negation of statement p is "not p," written as ~p.	Statement: $x = 3$ Negation: $x \neq 3$
polygon	98	A closed plane figure formed by three or more segments such that each segment intersects exactly two other segments only at their endpoints and no two segments with a common endpoint are collinear.	
quadrilateral	98	A four-sided polygon.	
theorem	110	A statement that has been proven.	Theorem 2-6-1: If two angles form a linear pair, then they are supplementary.
triangle	98	A three-sided polygon.	
truth value	82	A statement can have a truth value of true (T) or false (F).	If today is Sunday, then tomorrow is Monday. The truth value is true (T).

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2-1 Using Inductive Reasoning to Make Conjectures

Find the next term in each pattern.

1. 6, 12, 18, . . .

2. January, April, July, . . .

3. The table shows the score on a reaction time test given to five students in both the morning and afternoon. The lower scores indicate a faster reaction time. Use the table to make a conjecture about reaction times.

Student	Morning	Afternoon
Ann	2.4	1.9
Betsy	3.1	2.7
Carla	4.0	3.9
Denise	2.7	2.8
Ellen	2.2	2.0

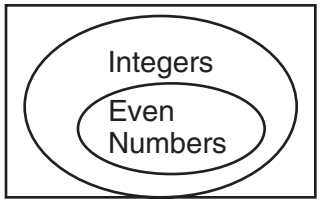
4. Show that the conjecture "If a number is a multiple of 5, then it is an odd number" is false by finding a counterexample.

2-2 Conditional Statements

5. Identify the hypothesis and conclusion of the conditional statement "Two angles whose sum is 90° are complementary angles".

Write a conditional statement from each of the following.

6.



7. An angle that measures 90° is a right angle.

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2-1 Using Inductive Reasoning to Make Conjectures

Find the next term in each pattern.

1. 6, 12, 18, . . .

24

2. January, April, July, . . .

October

3. The table shows the score on a reaction time test given to five students in both the morning and afternoon. The lower scores indicate a faster reaction time. Use the table to make a conjecture about reaction times.

Student	Morning	Afternoon
Ann	2.4	1.9
Betsy	3.1	2.7
Carla	4.0	3.9
Denise	2.7	2.8
Ellen	2.2	2.0

The scores for the afternoon test were lower, indicating a faster reaction time as compared to the morning test.

4. Show that the conjecture "If a number is a multiple of 5, then it is an odd number" is false by finding a counterexample.

(10, 20, 30) are counterexamples

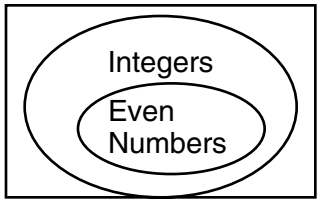
2-2 Conditional Statements

5. Identify the hypothesis and conclusion of the conditional statement "Two angles whose sum is 90° are complementary angles".

Hypothesis: Two angles whose sum is 90°.
Conclusion: The angles are complementary.

Write a conditional statement from each of the following.

6.



If a number is an even number, then it is an integer.

7. An angle that measures 90° is a right angle.

If an angle measurers 90°, then the angle is a right angle.

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Determine if each conditional is true. If false, give a counterexample.

8. If an angle has a measure of 90°, then it is an acute angle.

9. If $6x - 2 = 4x + 12$, then $x = 3$.

10. Write the converse, inverse, and contrapositive of the statement "If a number is divisible by 4, then it is an even number." Find the truth value of each.

converse:
truth value:
inverse:

truth value:
contrapositive:

truth value:

2-3 Using Deductive Reasoning to Verify Conjectures

11. Determine if the following conjecture is valid by the Law of Detachment.
Given: Nicholas can watch 30 minutes of television if he cleans his room first. Nicholas cleans his room.
Conjecture: Nicholas watches 30 minutes of television.

12. Determine if the following conjecture is valid by the Law of Syllogism.
Given: If a point A is on $\overline{MN}$, then it divides $\overline{MN}$ into $\overline{MA}$ and $\overline{AN}$. If $\overline{MA} \cong \overline{AN}$ then A is the midpoint of $\overline{MN}$.
Conjecture: If a point is on $\overline{MN}$, then A is the midpoint of $\overline{MN}$.

2-4 Biconditional Statements and Definitions

13. For the conditional "If two angles are complementary, then the sum of the measures is 90°," write the converse and a biconditional statement.

Converse:

Biconditional statement:

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Determine if each conditional is true. If false, give a counterexample.

8. If an angle has a measure of 90°, then it is an acute angle.

False, it is a right angle.

9. If $6x - 2 = 4x + 12$, then $x = 3$. **False, $x = 7$**

10. Write the converse, inverse, and contrapositive of the statement "If a number is divisible by 4, then it is an even number." Find the truth value of each.

converse: **If a number is an even number, then it is divisible by 4**
truth value: **F**
inverse: **If a number is not divisible by 4, then it is not an even number.**
truth value: **F**
contrapositive: **If a number is not an even number, then it is not divisible by 4.**
truth value: **T**

2-3 Using Deductive Reasoning to Verify Conjectures

11. Determine if the following conjecture is valid by the Law of Detachment.
Given: Nicholas can watch 30 minutes of television if he cleans his room first. Nicholas cleans his room.
Conjecture: Nicholas watches 30 minutes of television. **Valid**

12. Determine if the following conjecture is valid by the Law of Syllogism.
Given: If a point A is on $\overline{MN}$, then it divides $\overline{MN}$ into $\overline{MA}$ and $\overline{AN}$. If $\overline{MA} \cong \overline{AN}$ then A is the midpoint of $\overline{MN}$.
Conjecture: If a point is on $\overline{MN}$, then A is the midpoint of $\overline{MN}$.

No, it is not valid.

2-4 Biconditional Statements and Definitions

13. For the conditional "If two angles are complementary, then the sum of the measures is 90°," write the converse and a biconditional statement.

Converse: **If the sum of the measures of two angles is 90°, then the two angles are conplementary.**
Biconditional statement: **Two angles are complementary if and only if the sum of their measures is 90°.**

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14. Determine if the biconditional "A point divides a segment into two congruent segments if and only if the point is the midpoint of the segment," is true. If false, give a counterexample.

2-5 Algebraic Proof

Solve each equation. Write a justification for each step.

15. $m + 3 = -2$

16. $3m - 4 = 20$

17. $\frac{x}{-2} = -5$

Identify the property that justifies each statement.

18. $m\angle 1 \cong m\angle 2$, so $m\angle 1 + m\angle 3 = m\angle 2 = m\angle 3$

19. $\overline{MN} \cong \overline{PQ}$, so $\overline{PQ} \cong \overline{MN}$

20. $AB = CD$ and $CD = EF$, so $AB = EF$

21. $m\angle A = m\angle A$

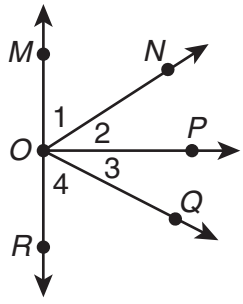
2-6 Geometric Proof

22. Fill in the blanks to complete the two-column proof.

Given: $m\angle MOP = m\angle ROP = 90°$
$\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

Proof:



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14. Determine if the biconditional "A point divides a segment into two congruent segments if and only if the point is the midpoint of the segment," is true. If false, give a counterexample.

True

2-5 Algebraic Proof

Solve each equation. Write a justification for each step.

15. $m + 3 = -2$

-3 -3 Sub. Prop. of Equality
$m = -5$ Simplify.

16. $3m - 4 = 20$

$+4$ $+4$, Add. Prop. of Equality
$3m = 24$, Simplify.
$\frac{3m}{3} = \frac{24}{3}$ Div. Prop. of Equality
$m = 8$ Simplify.

17. $\frac{x}{-2} = -5$

$-2\left(\frac{x}{-2}\right) = -5(-2)$, Mult. Prop. of Equality
$x = 10$ Simplify.

Identify the property that justifies each statement.

18. $m\angle 1 \cong m\angle 2$, so $m\angle 1 + m\angle 3 = m\angle 2 = m\angle 3$

Addition Property of Equality

19. $\overline{MN} \cong \overline{PQ}$, so $\overline{PQ} \cong \overline{MN}$

Symmetric Property of Congruence

20. $AB = CD$ and $CD = EF$, so $AB = EF$

Transitive Property of Equality

21. $m\angle A = m\angle A$

Reflexive Property of Equality

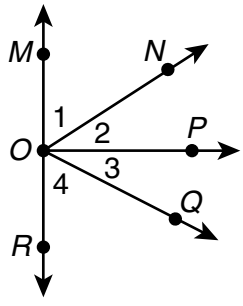
2-6 Geometric Proof

22. Fill in the blanks to complete the two-column proof.

Given: $m\angle MOP = m\angle ROP = 90°$
$\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

Proof:



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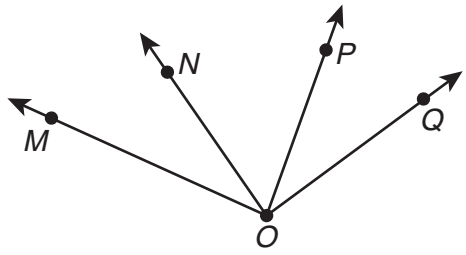
Statements	Reasons
1.	1. Given
2. $m\angle 1 = m\angle 4$	2.
3. $m\angle 1 + m\angle 2 = m\angle MOP$ $m\angle 3 + m\angle 4 = m\angle MOP$	3.
4.	4. Transitive Property of Equality
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	5.
6.	6. Subtraction Property of Equality

23. Use the given plan to write a two-column proof.



Given: $\angle MOP \cong \angle NOQ$

Prove: $\angle MON = \angle POQ$

Plan: By the definition of angle congruence, $m\angle MOP = m\angle NOQ$. Use the angle addition postulate to show that $m\angle MOP = m\angle MON + m\angle NOP$. Show a similar statement for $\angle NOQ$. Use the given fact to equate $m\angle MON + m\angle NOP$ and $m\angle POQ + m\angle NOP$. The subtraction property of equality allows you to show $m\angle MON = m\angle POQ$. Use the definition of congruent triangles to establish what needs to be proved.

Statements	Reasons

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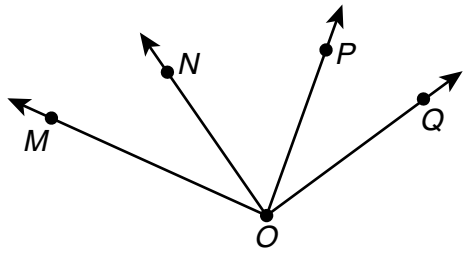
Statements	Reasons
1. $m\angle MOP = m\angle ROP = 90°$ $\angle 1 \cong \angle 4$	1. Given
2. $m\angle 1 = m\angle 4$	2. **Definition of Congruent Angles**
3. $m\angle 1 + m\angle 2 = m\angle MOP$ $m\angle 3 + m\angle 4 = m\angle MOP$	3. **Angle Addition Postulate**
4. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	4. Transitive Property of Equality
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	5. **Substitution**
6. $m\angle 2 = m\angle 3$	6. Subtraction Property of Equality

23. Use the given plan to write a two-column proof.



Given: $\angle MOP \cong \angle NOQ$

Prove: $\angle MON = \angle POQ$

Plan: By the definition of angle congruence, $m\angle MOP = m\angle NOQ$. Use the angle addition postulate to show that $m\angle MOP = m\angle MON + m\angle NOP$. Show a similar statement for $\angle NOQ$. Use the given fact to equate $m\angle MON + m\angle NOP$ and $m\angle POQ + m\angle NOP$. The subtraction property of equality allows you to show $m\angle MON = m\angle POQ$. Use the definition of congruent triangles to establish what needs to be proved.

Statements	Reasons
$\angle MOP \cong \angle NOQ$	**Given**
$m\angle MOP = m\angle NOQ$	**Definition of Congruent Angles**
$m\angle MON + m\angle NOP = m\angle MOP$ $m\angle POQ + m\angle NOP = m\angle NOQ$	**Angle Addition Postulate**
$m\angle MON + m\angle NOP = m\angle POQ + m\angle NOP$	**Transitive Property of Equality**
$m\angle MON = m\angle POQ$	**Subtraction Property of Equality**
$\angle MON \cong \angle POQ$	**Definition of Congruent Angles**

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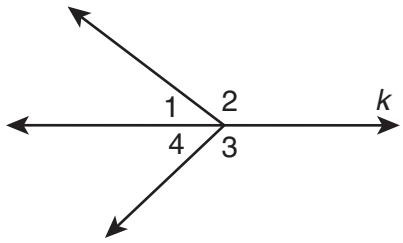
2-7 Flowchart and Paragraph Proofs



Use the given two-column proof to write the following.

Given: $\angle 1 \cong \angle 4$
Prove: $\angle 1$ is supplementary to $\angle 3$

Statements	Reasons
1. $\angle 1 \cong \angle 4$	1. Given
2. $m\angle 1 = m\angle 4$	2. Definition of Congruent Angles
3. $\angle 1$ & $\angle 2$ are supplementary. $\angle 3$ & $\angle 4$ are supplementary.	3. Linear Pair Theorem
4. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	4. Definition of Supplementary Angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	5. Transitive Property of Equality
6. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	6. Substitution Property of Equality
7. $m\angle 2 = m\angle 3$	7. Subtraction Property of Equality
8. $m\angle 1 + m\angle 3 = 180°$	8. Substitution Property of Equality
9. $\angle 1$ is supplementary to $\angle 3$.	9. Definition of Supplementary $\angle$s

24. a flowchart proof

25. a paragraph proof

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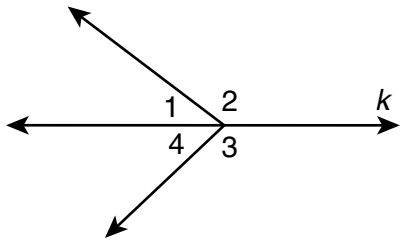
2-7 Flowchart and Paragraph Proofs

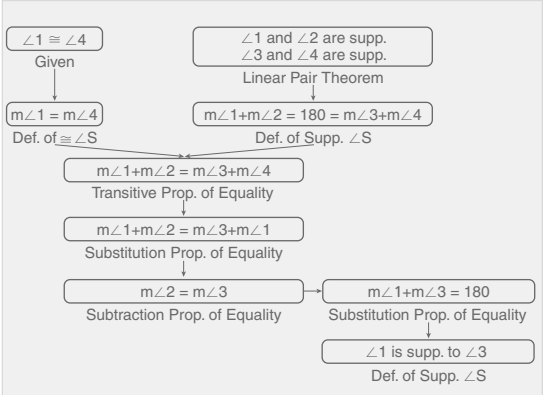


Use the given two-column proof to write the following.

Given: $\angle 1 \cong \angle 4$
Prove: $\angle 1$ is supplementary to $\angle 3$

Statements	Reasons
1. $\angle 1 \cong \angle 4$	1. Given
2. $m\angle 1 = m\angle 4$	2. Definition of Congruent Angles
3. $\angle 1$ & $\angle 2$ are supplementary. $\angle 3$ & $\angle 4$ are supplementary.	3. Linear Pair Theorem
4. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	4. Definition of Supplementary Angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	5. Transitive Property of Equality
6. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	6. Substitution Property of Equality
7. $m\angle 2 = m\angle 3$	7. Subtraction Property of Equality
8. $m\angle 1 + m\angle 3 = 180°$	8. Substitution Property of Equality
9. $\angle 1$ is supplementary to $\angle 3$.	9. Definition of Supplementary $\angle$s

24. a flowchart proof



25. a paragraph proof

The $m\angle 1 = m\angle 4$, as given and using the definition of congruent angles. Because of the Linear Pair theorem, $\angle 1$ and $\angle 2$ are supplementary, as are $\angle 3$ and $\angle 4$. Therefore, by the transitive property of equality, $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4 = 180°$. Additionally, since $m\angle 1 = m\angle 4$, $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$. By the Subtraction Property of Equality, $m\angle 2 = m\angle 3$. That implies that by substitution, $m\angle 1 + m\angle 3 = 180°$. This means that $\angle 1$ is supplementary to $\angle 3$ by definition of supplementary angles.

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Postulates and Theorems

Theorem 2-6-1 (Linear Pair Theorem) If two angles form a linear pair, then they are supplementary.

Theorem 2-6-2 (Congruent Supplements Theorem) If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.

Theorem 2-6-3 (Right Angle Congruence Theorem) All right angles are congruent.

Theorem 2-6-4 (Congruent Complements Theorem) If two angles are complementary to the same angle (or to two congruent angles), then the two angles are congruent.

Theorem 2-7-1 (Common Segments Theorem) If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

Theorem 2-7-2 (Vertical Angles Theorem) Vertical angles are congruent.

Theorem 2-7-3 If two congruent angles are supplementary, then each angle is a right angle.

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Postulates and Theorems

Theorem 2-6-1 (Linear Pair Theorem) If two angles form a linear pair, then they are supplementary.

Theorem 2-6-2 (Congruent Supplements Theorem) If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.

Theorem 2-6-3 (Right Angle Congruence Theorem) All right angles are congruent.

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Theorem 2-7-1 (Common Segments Theorem) If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

Theorem 2-7-2 (Vertical Angles Theorem) Vertical angles are congruent.

Theorem 2-7-3 If two congruent angles are supplementary, then each angle is a right angle.

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Big Ideas

Answer these questions to summarize the important concepts from Chapter 2 in your own words.

1. Explain how to verify whether a conjecture is true or false.

2. Explain the difference between the hypothesis and conclusion of a conditional statement.

3. What is a biconditional statement?

4. Explain the different types of geometric proofs.

For more review of Chapter 2:

- Complete the Chapter 2 Study Guide and Review on pages 130–133 of your textbook.
- Complete the Ready to Go On quizzes on pages 103 and 127 of your textbook.

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Big Ideas

Answer these questions to summarize the important concepts from Chapter 2 in your own words.

1. Explain how to verify whether a conjecture is true or false.

A conjecture is true if it can be proven through a series of logical, sequential steps. A conjecture is false if a counterexample can be found.

2. Explain the difference between the hypothesis and conclusion of a conditional statement.

The hypothesis is the part of the conditional statement following the word if.
The conclusion is the part of the conditional statement following the word then.
Example: A number is an integer, if it is a whole number.

3. What is a biconditional statement?

A biconditional statement is a statement that can be written in the form p if and only if q. For example: Two angles are congruent if and only if their measures are equal.

4. Explain the different types of geometric proofs.

There are three types of geometric proofs, two-column, flowchart and paragraph. A two-column proof lists statements and reasons. A flowchart proof uses boxes and arrows to show the structure of the proof. A paragraph proof presents the steps and matching reasons as sentences in a paragraph.

For more review of Chapter 2:

- Complete the Chapter 2 Study Guide and Review on pages 130–133 of your textbook.
- Complete the Ready to Go On quizzes on pages 103 and 127 of your textbook.

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